Damage Assessment of Structures Using Static Test Data

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An analytical method is presented for identifying the properties of structural elements from static test data. A set of static forces is applied to a set of degrees of freedom (DOF) and displacements are measured at another set of DOF. Utilizing this analytical method, the structural element stiffnesses are identified using the applied forces and measured displacements. This method is capable of determining changes in structural element stiffnesses, including element failure. The identified cross-sectional properties of the structural elements can be used for damage assessment and to determine the structure's load-carrying capacity.

Nomenclature

CN = condition number, $\lambda_{max}/\lambda_{min}$
DOF = degrees of freedom
DDOF = measured displacement DOF (1 by NMD)
$\{E(p)\}$ = error function vector; (NM by 1)
$[E(p)]$ = error function matrix; (NMD by NSF)
$\{f\}$ = applied forces vector; (1 by N)
$[F]$ = applied forces matrix; (N by N)
$[f_a]$ = force matrix corresponding to $[u_a]$; (NMD by NSF)
$[f_b]$ = force matrix corresponding to $[u_b]$; (NUD by NSF)
FDOF = applied force DOF; (NAF by 1)
ITER = number of iteractions. See Table 1b
$J(p + \Delta p)$ = scalar performance error function
$[K]$ = global stiffness matrix; (N by N)
$[k_{aa}]$ = submatrix of $[K]$. See Eq. (3); (NMD by NMD)
$[k_{ab}]$ = submatrix of $[K]$. See Eq. (3); (NMD by NUD)
$[k_{ba}]$ = submatrix of $[K]$. See Eq. (3); (NUD by NMD)
$[k_{bb}]$ = submatrix of $[K]$. See Eq. (3); (NUD by NUD)
LD = linear dependency between parameters
m_1 = measured displacement DOF (no forces applied)
m_2 = measured displacement and applied force DOF
m_3 = applied force DOF (no displacements measured
m_4 = no applied forces and no measured displacement DOF
N = $m_1 + m_2 + m_3 + m_4$ = total number of kinematic DOF
NAF = number of applied force DOF
NIM = number of independent measurements. See Eq. (15)
NM = number of measurements; (NMD by NSF)
NMD = number of measured displacements, $m_1 + m_2$
NSF = number of sets of applied forces, $m_2 + m_3$
NUD = number of unmeasured displacement DOF
NUP = number of unknown parameters
P_i = initial values of parameters. See Table 2
P_t = true values of parameters. See Table 2
PU = unknown parameters. See Tables 1a and 3a
$\{p\}$ = unknown parameter's values; (1 by NUP)
RSM = rank of the sensitivity matrix. See Table 1b
SVD = singular value decomposition
$[\underline{S}(p)]$ = sensitivity matrix; (NM by NUP)
$[S(p_j)]$ = sensitivity coefficients matrix; (NMD by NSF)
$\{u\}$ = displacements vector; (N by 1)
$[U]$ = displacements matrix; (N by N)
$[u_a]$ = measured displacements matrix; (NMD by NSF)
$[u_b]$ = unmeasured displacements matrix; (NUD by NSF)
$\{\Delta p\}$ = change in unknown parameters; (1 by NUP)
λ_{max} = maximum eigenvalue
λ_{min} = minimum eigenvalue

Introduction

PARAMETER identification of structural systems has become very important as researchers attempt to correlate changes in test data to changes in structural element properties. This way, deteriorations in cross-sectional properties of critical structures, such as aircraft, space stations, nuclear power plants, offshore drilling platforms, and bridges can be detected. The identified parameters can be used for damage assessment of these structures to improve their reliability and performance.

Extensive research has been performed in the area of parameter identification and is divided into two major categories: dynamic and static. Both techniques are based on the finite element method utilizing experimental test data. The intent of parameter identification is to adjust the parameters of the finite element model (FEM) to match the analytical and the measured data.

The literature on dynamic parameter identification is quite extensive. It is widely used in the aerospace industry, the earthquake engineering field, and on offshore oil platforms. Hart and Yao,[1] Baruch,[2] Berman,[3] Kabe,[4] Adelman and Haftka,[5] Beck,[6] Chen and Garba,[7] and Hajela and Soeiro[8] have compiled and summarized the work done in the area of dynamic parameter identification. Vibration testing is the most common method of dynamic parameter identification in structures, but there are still major limitations with this approach. Since dynamic parameter identification requires the use of the mass, stiffness, and damping properties, parameter identification is more complicated than a static method which only uses the stiffness properties of the structure.

Static parameter identification method can also be utilized for damage assessment of structures. Sheena et al.[9] presented a method on static parameter identification in which they optimized the difference between the theoretical and correct stiffness matrix subjected to equilibrium constraints. The method requires displacement measurements at all DOF used to define the FEM of the structure, which is not practical. A limited number of displacement measurements were used to artificially create the remaining displacement measurements using spline theorems of different orders. This introduces a major source of error into the stiffness matrix. Finally, they

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attempted to compute the stiffness matrix as a whole, not the cross-sectional properties of each element independently.

Sanayei and Nelson[10] presented a method to identify stiffness parameters for linear elastic structures subjected to static loads. Structural stiffnesses are identified at the element level using applied forces and measured displacements at a subset of DOF used to define the structural model. The method is limited to applying forces and measuring displacements at the same DOF. Sanayei and Scampoli[11] applied this technique and developed a computer program for parameter identification of a one-third scale reinforced concrete deck using incomplete static test data.

Clark[12] presented an algorithm similar to that presented in Ref. 10 for determining member stiffnesses from complete and incomplete sets of measured displacements. Clark expanded the aforementioned identification algorithm to estimate member stiffnesses using modal displacements data for stiffness identification.

Hajela and Soeiro[8] presented a state-of-the-art paper on damage detection and classified the identification techniques into the equation error approach, the output error approach, and the minimum deviation approach. This paper compared these methods for their effectiveness to minimize the difference between the analytical model and measured data. These methods covered both static and dynamic parameter identification.

The parameter identification method presented in this paper utilizes static applied loads at one subset of DOF and measured displacements at another subset of DOF to detect damage in structural elements. Even though structural damage is a nonlinear type of behavior, this method will use loads of small magnitudes that will cause structures to behave only in their linear elastic range. The purpose of the nondestructive testing is to identify the equivalent cross-sectional properties of each component. A structural component is considered damaged if any of its cross-sectional properties has been substantially reduced.

In the parameter identification algorithm, the finite element model, the topology of the structure, the behavior of the elements, and connections are specified at the outset. The unknown features of the structural model are some or all of the parameters used to describe the structural properties, e.g., cross section area, moment of inertia. Static loads are applied and displacements are measured at limited and predefined points on the structure.

Forces and displacements are simulated including no measurement or modeling errors. Onipede[13] has investigated the effect of measurement errors on the identifed parameters. For each system there exists a linear range of input-output error behavior. Although errors in the identified parameters are always magnified, if the input error is small enough to utilize the linear input-output error behavior range, it is possible to use the identified parameters for damage assessment. If the finite element modeling errors are very small, they can be considered as a part of the measurement errors. If the modeling errors are significant, the finite element model should be adjusted before a successful parameter identification can be performed.

The FEM is used to simulate a measured response to the same applied test loads by using the true values of the parameters. For a damaged structure the true values of some of the parameters are different from their initial values. Given the initial values (prior to damage) differing from the true values (after the damage), analytical displacements are evaluated for the same subset of DOF using the same FEM. Obviously, the simulated measured displacements will be different from those of the analytical FEM.

An iterative procedure is used to minimize automatically the difference between the measured and the theoretical response. A sensitivity analysis is used at the heart of this iterative method to identify directly the structural element parameters. Throughout the iterations, all the properties of the stiffness matrix such as element connectivity, positive definiteness, symmetry, equilibrium, and bandedness are automatically preserved.

Many of the traditional methods are capable of detecting structural deterioration, but the problem of predicting the effect of the deterioration on the performance of the structure remains. The analytical method of parameter identification has the following benefits: 1) structural stiffness properties are identifed at the structural element level; 2) overall load capacity of a structure can be easily determined from the identified parameters; 3) only a limited number of static force and displacement measurements are required; 4) static testing is simple, rapid, economical, and does not require the use of sophisticated equipment; 5) it is capable of detecting damage in various structural materials including steel, concrete, aluminum, and wood; 6) by conducting tests over certain time intervals the remaining life of structural elements can be estimated, thus the remaining life of the structure can be predicted.

Parameter Identification

The parameter identification method presented here is an extension of the work done by Sanayei and Nelson.[10] This work was limited to applying static forces and measuring displacements at the same DOF. *The objective of this paper is to present a method of parameter identification at the element level by applying sets of static forces at one subset of DOF and measuring displacements at another subset of DOF. The applied force DOF and measured displacement DOF may completely overlap, partially overlap, or not overlap at all.* The measured DOF are the same ones used to define the finite element model of a linear elastic structure. The identified changes in the unknown parameters (e.g., area, moment of inertia) are used to determine whether there is any degradation or failure of the critical components of various structures.

The finite element method is based on the force-displacement relationship,

$$\{f\} = [K]\{u\} \tag{1}$$

To perform damage assessment the unknown parameters are selected to be identified. The rest of the parameters are assumed to be known with a high level of confidence. There are two different sets of DOF for measurement purposes: the applied forces (FDOF) and the measured displacements (DDOF). These two sets of DOF may or may not overlap. NSF sets of forces are applied at FDOF, one set at a time, and NSF sets of displacements are measured at DDOF. Each set of force should be neither equal to any other set nor a linear combination of the previous sets of applied forces. These sets of applied forces and measured displacements are concatenated horizontally into a force matrix $[F]$ and a displacement matrix $[U]$ as shown below:

$$[F] = [K][U] \tag{2}$$

In fact, not all displacements in $[U]$ need to be measured. Therefore, Eq. (2) is partitioned into $[u_a]$ and $[u_b]$, which are the measured and unmeasured displacements.

$$\left[\begin{array}{c} f_a \\ \hline f_b \end{array}\right] = \left[\begin{array}{c|c} k_{aa} & k_{ab} \\ \hline k_{ba} & k_{bb} \end{array}\right] \left[\begin{array}{c} u_a \\ \hline u_b \end{array}\right] \tag{3}$$

The matrix of unmeasured displacements $[u_b]$ is condensed out and Eq. (3) is reduced to

$$[f_a] = \Big[[k_{aa}] - [k_{ab}][k_{bb}]^{-1}[k_{ba}]\Big][u_a] + [k_{ab}][k_{bb}]^{-1}[f_b] \tag{4}$$

Matrices $[f_a]$, $[f_b]$, and $[u_a]$ are obtained from the test data. The analytical stiffness matrices $[k_{aa}]$, $[k_{ab}]$, $[k_{bb}]$ and $[k_{ba}]$ are functions of cross-sectional parameters $\{p\}$. Equation (4) is a nonlinear function of the stiffness parameters due to inversion of $[k_{bb}]$.

In order to identify the stiffness parameters, an error matrix $[E(p)]$ is defined,

$$[E(p)] = \left[[k_{aa}] - [k_{ab}][k_{bb}]^{-1}[k_{ba}] \right][u_a] + [k_{ab}][k_{bb}]^{-1}[f_b] - [f_a] \tag{5}$$

If the stiffness parameters $\{p\}$ are correct, then $[E(p)]$ will be zero, otherwise $[E(p)]$ will not be zero. Next, all elements of $[E(p)]$ are placed into an error vector, $\{E(p)\}$, of size NM by 1. To adjust the parameters $\{p\}$ in $\{E(p)\}$, a first-order Taylor series expansion is used to linearize the vector $\{E(p)\}$ that is a nonlinear function of the parameters:

$$\{E(p + \Delta p)\} \approx \{E(p)\} + [S(p)]\{\Delta p\} \tag{6}$$

where,

$$[S(p)] = \left[\frac{\partial\{E(p)\}}{\partial\{p\}} \right] \tag{7}$$

In order to analytically evaluate the sensitivity matrix $[S(p)]$ in Eq. (7), the error matrix $[E(p)]$ in Eq. (5) is differentiated with respect to each parameter.

$$\begin{aligned}[\bar{S}(p_j)] = {} & \left[\frac{\partial[k_{aa}]}{\partial P_j} - \frac{\partial[k_{ab}]}{\partial P_j}[k_{bb}]^{-1}[k_{ba}] \right. \\ & - [k_{ab}][k_{bb}]^{-1}\frac{\partial[k_{ba}]}{\partial P_j} \\ & \left. + [k_{ab}][k_{bb}]^{-1}\frac{\partial[k_{bb}]}{\partial P_j}[k_{bb}]^{-1}[k_{ba}] \right][u_a] \\ & + \left[\frac{\partial[k_{ab}]}{\partial P_j}[k_{bb}]^{-1} - [k_{ab}][k_{bb}]^{-1}\frac{\partial[k_{bb}]}{\partial P_j}[k_{bb}]^{-1} \right][f_b] \end{aligned} \tag{8}$$

The sensitivity coefficient in Eq. (8) is evaluated for $j = 1$ to NUP, where NUP is the number of unknown parameters. Similar to $[E(p)]$, the elements of $[\bar{S}(p_j]$ are assembled into a vector of size NM. These vectors are horizontally concatenated for $j = 1$ to NSF to form the sensitivity matrix $[S(p)]$ of size NM by NUP.

A scalar performance error function is defined as,

$$J(p + \Delta p) = \{E(p + \Delta P)\}^T\{E(p + \Delta p)\} \tag{9}$$

The stiffness parameters are obtained by minimizing $[J(p + \Delta p)]$ with respect to the unknown parameters $\{p\}$.

$$\frac{\partial J(p + \Delta p)}{\partial\{p\}} = \{0\} \tag{10}$$

Equation (11) is derived from Eqs. (6), (9), and (10),

$$[S(p)]^T[E(p) + [S(p)]\{\Delta p\}] = \{0\} \tag{11}$$

Since sets of forces are applied and sets of displacements are measured, the number of independent measurement (NIM) may be different from the number of unknown parameters (NUP). If NUP is greater than NIM, a unique solution of Eq. (11) does not exist. If NUP is equal to NIM, a direct inversion can be used to solve Eq. (11).

$$\{\Delta p\} = -[S(p)]^{-1}\{E(p)\}^i \tag{12}$$

If NUP is less than NIM, then $[S(p)]$ will not be a square matrix. The method of least squares is utilized to compute the unknown parameters for each iteration.‡

$$\{\Delta p\} = -\left[[S(p)]^T[S(p)] \right]^{-1}[S(p)]^T\{E(p)\}^i \tag{13}$$

Equation (12) or (13) is used to set up an iterative procedure for parameter identification as,

$$\{p\}^{i+1} = \{p\}^i + \{\Delta p\} \tag{14}$$

Additional Concepts in Parameter Identification

In order to use the parameter identification method successfully and efficiently, several guidelines that are useful in making a physical interpretation of the mathematical phenomena observed during execution of the algorithm are briefly described.

Identifiability of Structural Parameters

In order to identify a unique set of parameters from a given set of measurements, the number of independent measurements must be greater than or equal to the number of unknown parameters. If the aforementioned condition does not hold, there may exist an infinite number of values of parameters that satisfy these measurements.

Number of Independent Measurements

The number of independent measurements (NIM) is the total number of measurements minus the number of redundant measurements due to symmetry in the normalized matrix of measured displacements, $[U_a]$. The number of independent measurements for single load cases is

$$NIM = (m_1 + m_2)(m_2 + m_3) - \tfrac{1}{2} m_2(m_2 - 1) \tag{15}$$

For the special case of complete overlap between FDOF and DDOF, Eq. (15) reduces to

$$NIM = \tfrac{1}{2} m_2(m_2 + 1) \tag{16}$$

Linear Dependencies in the Sensitivity Matrix

The existence of a solution to Eq. (11) is determined by using the elementary row operations to reduce the augmented matrix $[S(p)]$, $\{E(p)\}$ to the row echelon form. The number of nonzero rows in a row echelon form of a matrix is its rank. The rank of the sensitivity matrix is equal to NIM. Also in each nonzero row of the row echelon form of $[S(p)]$, $\{E(p)\}$ the existence of more than one nonzero element represents a linear dependency between parameters corresponding to these columns of $[S(p)]$. Linear dependencies reduce the capability of the parameter identification algorithm presented. If a group of parameters is linearly dependent, only one can be identified uniquely when the rest of them are known.

One of the reasons for linear dependencies in the sensitivity matrix is that each type of element is capable of transferring a limited number of independent pieces of information. For example, two-dimensional beam elements in bending are capable of transferring a maximum of three pieces of information across the element: pure shear, pure bending, and the interaction of shear and bending. A truss element is capable of only transferring one piece of information. The sensitivity matrix will also contain a rank deficiency if any of the structural elements (parameters) is not stressed by any of the forces applied.

If the sensitivity matrix is ill-conditioned, it may not be inverted with a high precision. The program can use the singular value decomposition (SVD) method to solve an ill-con-

‡If the sensitivity matrix is singular or ill-conditioned, singular value decomposition may be used to compute some of the unknown parameters. This method is more computation intensive.

ditioned or a rank-deficient system of equations instead of using direct inversion or the least-squares method. Using SVD it is possible to identify the linearly independent parameters.

Criteria for Convergence of the Algorithm

Various criteria are used to check the algorithm for convergence. These are: change in the scalar error function, $J(p)$; changes in the error matrix, $E(p)$; changes in the parameters, $\{\Delta P\}$; relative change in the parameters compared to their initial value, $\{\Delta P\}/\{P_i\}$.

Tolerance limits are set for each criteria. When any of the limits are reached the algorithm is considered to have converged. These limits can be used to control the desired accuracy in the identified parameters. In addition, upper and lower bound constraints are defined for all unknown parameters for minimization.

Example 1: Two-Story Trapezoidal Truss Model

A two-story trapezoidal truss is illustrated in Fig. 1. It is used to demonstrate the parameter identification algorithm relating physical behavior (simulated) of the structure to the analytical model (FEM). Since this structural model consists of axial members only, each element has one parameter, area. The physical geometries and the material properties are given as follows:

Modulus of elasticity of all elements: 30,000 ksi (206.8 GPa)
Initial value of all parameters, P_i (area): 5.0 in.^{2} (32.26 cm^2)
True value of all parameters, P_t (area): 3.0 in.^{2} (19.35 cm^2)

Ten different cases are evaluated as shown in the Tables 1a and 1b. The symbols used in the tables are defined under "Nomenclature" at the beginning of the paper. In each case, a 100 kip (445 kN) force is applied to each FDOF selected (one at a time) and a set of displacements is measured at the selected DDOF (one set at a time). In the first five cases all 10 parameters are assumed to be unknown. The first two cases are cases of complete overlap between FDOF and DDOF. Cases 3, 4, and 5 are cases of partial overlap between FDOF and DDOF. In this paper, when a case is declared "converged," it means that the parameter estimates are evaluated successfully and contain no error (identical to the true values of parameters used to simulate the measurements).

In case 1, forces are applied and displacements are measured at all eight DOF. All 10 parameters are assumed to be unknown, hence at least 10 independent measurements are required. There are 36 (NIM = 8(8 + 1)/2) independent measurements due to measuring all force and displacement DOF. The excessive number of independent measurements leads to rapid convergence of the algorithm on the true values of the parameters.

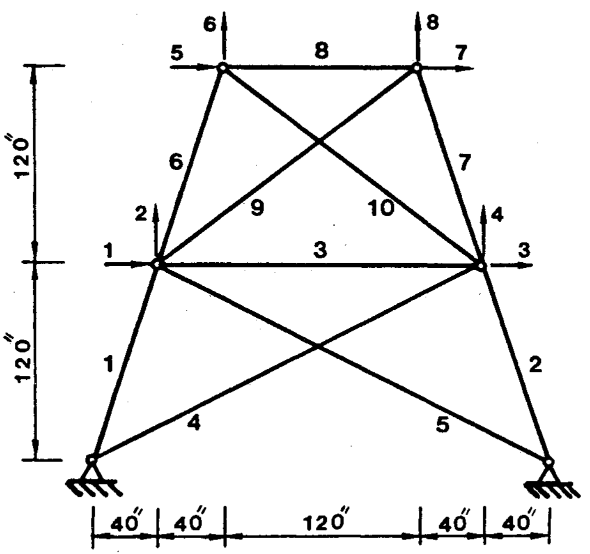

Fig. 1 Two-story trapezoidal truss model.

Table 1a Parameter identification of the truss model

Case	FDOF	DDOF	NUP	PU	NIM
1	1–8	1–8	10	1–10	36
2	5–8	5–8	10	1–10	10
3	5–8	1,2,5–8	10	1–10	18
4	1–4	1–8	10	1–10	26
5	5,8	1–8	10	1–10	15
6	1–4	1–4	5	1–5	10
7	7	1–8	8	1,2,4,6–10	8
8	7,8	1–6	7	1–6,10	12
9	5–8	1–4	5	1–5	16
10	5,6	1–4	5	1–5	8

Table 1b Parameter identification of the truss model

Case	CN	λ_{min}	RSM	ITER	Results
1	7.553	554.9785	10	2	Converged
2	2562.2	0.3031	10	2	Converged
3	413.6295	8.6709	10	2	Converged
4	4.6E+16	2.3E−14	5	5	Converged (SVD)
5	69.2452	37.7808	10	2	Converged
6	2.4676	432.1909	5	2	Converged
7	139.1631	8.8614	8	2	Converged
8	46.8535	29.4115	7	2	Converged
9	22.2614	91.4290	5	2	Converged
10	44.2100	29.5623	5	2	Converged

In case 2, forces are applied and displacements are measured at the upper four DOF. In this case there are 10 unknown parameters and 10 independent measurements (NIM = 4(4 + 1)/2). This case took just two iterations to converge, even though only four of the eight DOF are measured.

Cases 3, 4, and 5 contain partial overlap between FDOF and DDOF. Case 3 is similar to case 2 except that an additional two DDOF are measured. The addition of these two DDOF significantly reduces the condition number and increases the smallest eigenvalue. In case 4, forces are applied to DOF 1 to 4 and displacements are measured at all eight DOF. There are linear dependencies in some of the unknown parameters so SVD is used in the algorithm and only parameters 1 through 5 are identified. In case 5 by applying only two forces to DOF 5 and 8 and measuring the same displacement DOF as case 4, the algorithm identifies all the parameters. This is because when forces are applied to DOF 5 and 8 all the elements in the structure are stressed significantly. Since displacements are measured at all DOF, the algorithm converges in only two iterations.

In cases 6 to 10, only some of the parameters are to be identified. It is assumed that the true values of the other parameters are known with a high degree of confidence. The known parameters use the true values, whereas the unknown parameters use the initial values to start the algorithm. Case 6 is another case of complete overlap, but only the five lower level elements are unknown (i.e., parameters 1 to 5). This case converges in only two iterations.

Case 7 contains a partial overlap between FDOF and DDOF. This represents a realistic situation where a limited number of forces are applied and several displacement measurements are taken. Even though there is only one applied force, all the elements with unknown parameters are stressed and all displacements are measured. Since all DOF are measured, the program converges in two iterations.

Cases 8, 9, and 10 represent situations of no overlap between FDOF and DDOF. In case 8 by applying forces to two DOF and measuring six other DOF, 12 independent measurements were made (NIM = 2 × 6). In this case, seven unknown parameters are identified in two iterations. By applying forces to DOF 5 to 8 and measuring displacements at DOF 1 to 4, parameters 1 to 5 are identified in case 9. In case 10 by applying forces to two DOF and measuring displacements at four DOF, five unknown parameters are identified in two iterations.

All converged cases identified the true values of the unknown parameters accurately. If all displacements are measured, the algorithm converges in two iterations. The algorithm actually identifies the parameters in one iteration, but to verify convergence a second iteration is required. If fewer measurements are made, it will take more than two iterations for the algorithm to converge.

Example 2: Frame Model

A two-dimensional frame is illustrated in Fig. 2. It is used to demonstrate the parameter identification algorithm relating physical behavior (simulated) of the structure to the analytical model. Modulus of elasticity of all elements is assumed 30,000 ksi (206.8 GPa). In each case, a 100 kip (445 kN) force is applied to each FDOF selected (one at a time) and a set of displacements is measured at the selected DDOF (one set at a time). Since this is a two-dimensional frame, each element has two parameters, area and moment of inertia. The initial values and true values of the parameters are given in Table 2. These parameters are numbered 1 to 12, corresponding to elements 1 to 6 (e.g., parameters 3 and 4 are the area and moment of inertia of element number 2).

Ten different cases are presented as shown in Tables 3a and 3b. In the first three cases, all 12 parameters are assumed to be unknown. Cases 1, 4, 5, and 6 are examples of complete overlap between FDOF and DDOF. Cases 2, 3, and 7 are examples of partial overlap. Cases 8 and 9 represent no overlap between FDOF and DDOF. Case 10 illustrates the use of multiple applied forces per load case.

In case 1, forces are applied and displacements are measured at all FDOF and DDOF. When all DOF are measured, Eq. (5) reduces to a linear function of the unknown parameters and Eq. (11) simplifies to a linear algebraic system of equations. There are 66 (NIM = 11 × 11 − 11(11 − 1)/2) independent measurements and only 12 unknown parameters. The rapid convergence is achieved due to excessive measurements and solution of a linear system of equations. Since there are no errors in the measurements, there are no errors in the identified parameters.

In case 2, there is a partial overlap between the measured DOF. There is an overlap of four between FDOF and DDOF. In this case there are 12 unknown parameters and 19 independent measurements (NIM = 5 × 5 − 4(4 − 1)/2). This case converged accurately in four iterations. Case 3 has two applied forces and seven measured displacements. There is an overlap of two DOF between FDOF and DDOF. This induces 13 independent measurements, which is more than the 12 required for identification purposes. Convergence is achieved in eight iterations.

In case 4 there is a complete overlap between FDOF and DDOF. It is assumed that parameters 1 and 2 are known and the other 10 parameters are assumed unknown. Since there

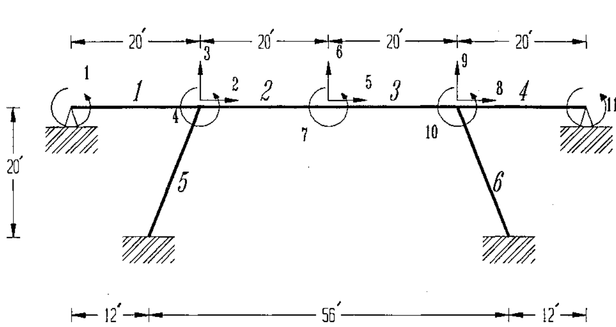

Fig. 2 Frame model.

are 10 independent measurements and 10 unknowns, instead of the least-squares method, direct inversion is used to find the unknown parameters and convergence is achieved in eight iterations.

For cases 5 to 10, parameters associated with elements 1, 4, 5, and 6 are known with a high confidence, whereas parameters associated with elements 2 and 3 are unknown. These cases can represent a situation of local damage. Case 5 represents a case of complete overlap between FDOF and DDOF. Even though there are six independent measurements, the algorithm is unable to identify all four unknown parameters. Only DOF 8 provides information about axial deformation. The parameters associated with axial deformation (i.e., A2 and A3) are linearly dependent and therefore SVD is used to accurately determine I2 and I3, while A2 and A3 are not found. Case 6 is similar to case 5 except that one DOF associated with bending only is switched with one DOF associated with axial deformation. In this case all four unknown parameters are identified in four iterations. There is a significant change in the condition numbers and the smallest eigenvalues of cases 5 and 6.

Case 7 is a case of partial overlap. Applying forces to only two of the three DDOF, all four unknown parameters are identified. Compared to case 6, it uses less information to identify the unknown parameters, but takes more iterations to converge.

Cases 8 and 9 represent situations of no overlap between the FDOF and DDOF. In case 8 seven DOF are measured, while two DOF have forces applied to them. It takes only three iterations to identify the unknown parameters. By reducing the number of DDOF to six, it takes four iterations for the algorithm to identify the unknown parameters.

Case 10 is similar to case 7, except that forces are applied to two DOF in each set of applied forces. It is still a case with two sets of FDOF and three sets of DDOF. Multiple applied load capability is beneficial to create more complex load patterns for various tests in an attempt to stress all components of the structure for a successful parameter identification. Case 10 also converged on the true values of the parameters in six iterations. In this paper, all cases labeled "converged" identified the true values of the unknown parameters accurately.

In addition to the examples presented, other two-dimensional and three-dimensional trusses and frames were ex-

Table 2 Initial values and true values of parameters

		P_i		P_t			P_i		P_t	
Element	PU	in.^{2}	cm^2	in.^{2}	cm^2	PU	in.^{4}	cm^4	in.^{4}	cm^4
1	A1	96	619	94	606	I1	1824	75921	1823	75879
2	A2	96	619	36	232	I2	1824	75921	124	5161
3	A3	96	619	46	297	I3	1824	75921	824	34297
4	A4	96	619	78	503	I4	1824	75921	1756	73090
5	A5	225	1452	165	1065	I5	50625	2107172	10625	442246
6	A6	225	1452	215	1387	I6	50625	2107172	50615	2106755

Table 3a Parameter identification of the frame model

Case	FDOF	DDOF	NUP	PU	NIM
1	1–11	1–11	12	1–12	66
2	2,3,5,8,9	2,3,6,8,9	12	1–12	19
3	5,6	2,3,5–9	12	1–12	13
4	3,5,8,9	3,5,8,9	10	3–12	10
5	3,6,8	3,6,8	4	3–6	6
6	3,5,8	3,5,8	4	3–6	6
7	5,6	2,5,6	4	3–6	5
8	3,9	2,4–8,10	4	3–6	14
9	3,9	2,4–	4	3–6	12
10	3&5,6&9	6,8,10	4	3–6	6
		2,5,6			

Table 3b Parameter identification of the frame model

Case	CN	λ_{min}	RSM	ITER	Results
1	1.2E+08	1.9E−05	12	2	Converged
2	1.1E+12	1.6E−12	12	4	Converged
3	2.2E+13	6.3E−11	12	8	Converged
4	3.4E+13	9.3E−14	10	6	Converged
5	9.1E+16	3.4E−18	3	10	LD
6	8.0E+08	3.8E−09	4	4	Converged
7	2490.3	1.7E−03	4	6	Converged
8	1423.2	1.1E−04	4	3	Converged
9	1851.9	8.3E−05	4	4	Converged
10	3464.7	1.6E−03	4	6	Converged

amined. A computer program utilizing this method for parameter identification using static forces and displacement measurements has been successfully developed and applied.

Conclusions

The proposed parameter identification method used a limited number of simulated applied forces and measured displacements and identified some or all of the structural parameters. Since the simulated measurements included no measurement error, the identified parameters contained no errors.

In this method, forces can be applied and displacements can be measured at two different sets of DOF. These two sets of DOF may completely overlap, partially overlap, or not overlap. The success of this iterative method and the rate of convergence is case dependent. It is a function of the topology of the structure, number of independent measurements, and the parameters to be identified. From the results of the examples presented, it is concluded that the algorithm converges faster when more independent measurements are made.

Another important factor is to select the most sensitive DOF associated with the unknown parameters. In the frame example, the effect of measuring more sensitive DOF is clear from cases 5 and 6, where three DOF are used in each case. In case 5 only two of the unknown parameters are identified, whereas in case 6 all four unknowns are identified. By selecting a more sensitive DOF (five instead of six), the algorithm converged and the parameters were accurately identified.

The proposed parameter identification method using simulated measured static displacements was successful in performing damage assessment at the element level, and the results are encouraging and positive. Further research is in progress to investigate the impact of measurement and modeling errors on the identified parameters. In addition, laboratory tests on small-scale structures are being performed to validate the proposed parameter identification method.

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